## Cornerstone Electronics Technology and Robotics I Week 15 Combination Circuits (Series-Parallel Circuits)

- Administration:
o Prayer
o Turn in quiz
- Electricity and Electronics, Chapter 8, Introduction:
o In this lesson, we will learn to analyze circuits with resistors in both series and parallel together and compute the total resistance (equivalent resistance) for those circuits. From there, we can calculate the voltage drop and current for each resistor in the circuit.
o Series-parallel circuits are also called combination circuits.
o Examples:


Figure 1 - Examples of Combination Circuits
o See applets:

- http://www.falstad.com/circuit/e-resistors.html
- http://webphysics.davidson.edu/physlet resources/bu semester2/ c10 combo.html
o Regardless of the complexity of a series-parallel circuit, it can be solved when broken down into steps of individual series and parallel circuits.
o Review:
- Four equations are used to solve series resistor circuits. They are:

$$
\begin{aligned}
& R_{T}=R_{1}+R_{2}+R_{3}+\ldots+R_{N} \\
& V_{T}=V_{1}+V_{2}+V_{3}+\ldots+V_{N} \\
& I_{T}=I_{1}=I_{2}=I_{3}=\ldots I_{N} \\
& V=I \times R
\end{aligned}
$$

$\mathrm{V}=\mathrm{I} \times \mathrm{R}$ can be applied to the total circuit $\left(\mathrm{V}_{\mathrm{T}}=\mathrm{I}_{\mathrm{T}} \times \mathrm{R}_{\mathrm{T}}\right)$ and to individual resistors $\left(V_{1}=I_{1} \times R_{1}\right)$.

- Four equations are used to solve parallel resistor circuits. They are:

$$
\begin{aligned}
& 1 / R_{T}=1 / R_{1}+1 / R_{2}+1 / R_{3}+\ldots \ldots \ldots .+1 / R_{N} \\
& V_{T}=V_{1}=V_{2}=V_{3}=\ldots . . V_{N} \\
& I_{T}=I_{1}+I_{2}+I_{3}+\ldots+I_{N} \\
& V=I \times R
\end{aligned}
$$

See Lesson 14, Parallel Circuits for details. Link: http://cornerstonerobotics.org/curriculum/lessons year1/ER\%20Week14,\%20Parallel \%20Circuits.pdf
o Power in Combination Circuits: The total power in a combination circuit is equal to the sum of the individual powers.
o The variables used in Ohm's Law equations must be common to the same two points in the circuit under consideration. This rule cannot be overemphasized. This is especially important in series-parallel combination circuits where nearby components may have different values for both voltage drop and current.

- Electricity and Electronics, Section 8.1, Solving a Complex Circuit:
o The goal of series-parallel resistor circuit analysis is to be able to determine all voltage drops, currents, and power dissipations in a circuit.
o Contracting the Circuit: Finding the total resistance and total current: In this section, you will combine series and parallel resistors in a step by step process until the entire circuit is reduced to one equivalent resistor.
- Step 1: Assess which resistors in a circuit are connected together in simple series or simple parallel.
- Step 2: Re-draw the circuit, replacing each of those series or parallel resistor combinations identified in step 1 with a single, equivalent-value resistor.
- Step 3: Repeat Steps 1 and 2 until the entire circuit is reduced to one equivalent resistor.
- Step 4: Find the total current for the circuit.
o Expanding the Circuit: In this section, you will proceed step-by-step from the simplified version of the circuit back to its original form, plugging in values of voltage and current where appropriate until all values of voltage and current are known.
- Step 5: Taking total voltage and total current values, go back to last step in the circuit reduction process and insert those values where applicable.
- Step 6: From known resistances, total voltage, and total current values from step 5 , use Ohm's Law to calculate unknown values (voltage or current)
- Step 7: Repeat steps 5 and 6 until all values for voltage and current are known in the original circuit configuration. Essentially, you will proceed step-by-step from the simplified version of the circuit back to its original, complex form, plugging in values of voltage and current where appropriate until all values of voltage and current are known.
- Step 8: Calculate power dissipations from known voltage, current, and/or resistance values.

Contracting a combination circuit to determine the one equivalent resistor:

Expanding a combination circuit to determine the circuit currents and voltages:


Figure 2 - Example of Contracting and Expanding a Combination Circuit

Example 1: Parallel resistors $R_{1}$ and $R_{2}$ are in series with resistor $R_{3}$. See Figure 3.
(Note: In the following examples, when you encounter a resistor labeled $R_{1,2}$, it is an equivalent resistor for resistors $R_{1}$ and $R_{2}$. $R_{1,2,3,4,5}$ is the equivalent resistor for resistors $R_{1}, R_{2}, R_{3}$, $\mathrm{R}_{4}$, and $\mathrm{R}_{5}$. Similarly, $\mathrm{V}_{4,5}$ is the voltage drop across the equivalent resistor $\mathrm{R}_{4,5}$.)


Figure 3
Solve for the all of the unknowns in Table 1.

|  | Resistance | Voltage | Current |
| :---: | :---: | :---: | :---: |
| $\mathrm{R}_{1}$ | $100 \Omega$ | $\mathbf{V}_{1}$ | $\mathbf{I}_{\mathbf{1}}$ |
| $\mathrm{R}_{2}$ | $100 \Omega$ | $\mathbf{V}_{2}$ | $\mathbf{I}_{\mathbf{2}}$ |
| $\mathrm{R}_{3}$ | $50 \Omega$ | $\mathbf{V}_{3}$ | $\mathrm{I}_{3}$ |
| Total | $\mathbf{R}_{\mathrm{T}}$ | 50 V | $\mathbf{I}_{\mathrm{T}}$ |

## Table 1

o Contracting the Circuit: Finding the Total Resistance, $\mathrm{R}_{\mathrm{T}}$ and Total Current, $\mathrm{I}_{\mathrm{T}}$ :

- Step 1: Assess which resistors in a circuit are connected together in simple series or simple parallel. In our circuit, $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ are in a simple parallel configuration.
- Step 2: Re-draw the circuit, (Figure 4), replacing each of those series or parallel resistor combinations identified in Step 1 with a single, equivalent-value resistor. In this circuit, $R_{1}$ and $R_{2}$ are replaced with $\mathrm{R}_{1,2}$.


Figure 4

$$
\begin{aligned}
& \mathrm{R}_{1,2}=\mathrm{R} / \mathrm{N} \\
& \mathrm{R}_{1,2}=100 \Omega / 2 \\
& \mathrm{R}_{1,2}=50 \Omega
\end{aligned}
$$

- Step 3: Repeat Steps 1 and 2 until the entire circuit is reduced to one equivalent resistor. (Step 1) - $\mathrm{R}_{1,2}$ and $\mathrm{R}_{3}$ are in a simple series configuration. (Step 2) - Re-draw the circuit, (Figure 5), with $R_{1,2,3}$ replacing $R_{1,2}$ and $R_{3}$. Enter $R_{T},\left(R_{1,2,3}\right)$, into the solution table (Table 2).


Figure 5

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{T}}=\mathrm{R}_{1,2,3} \\
& \mathrm{R}_{\mathrm{T}}=\mathrm{R}_{1,2}+\mathrm{R}_{3} \\
& \mathrm{R}_{\mathrm{T}}=50 \Omega+50 \Omega \\
& \mathrm{R}_{\mathrm{T}}=100 \Omega
\end{aligned}
$$

|  | Resistance | Voltage | Current |
| :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | V 1 | I 1 |
| R 2 | $\mathrm{R} 2=100 \Omega$ | V 2 | I 2 |
| R 3 | $\mathrm{R} 3=50 \Omega$ | V 3 | I 3 |
| Total | $\mathrm{R}=100 \Omega$ | $\mathrm{~V}_{\mathrm{T}}=50 \mathrm{~V}$ | $\mathrm{I}_{\mathrm{T}}$ |

## Table 2

- Step 4: Find the total current for the circuit.

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{T}}=\mathrm{V}_{\mathrm{T}} / \mathrm{R}_{\mathrm{T}} \\
& \mathrm{I}_{\mathrm{T}}=50 \mathrm{~V} / 100 \Omega \\
& \mathrm{I}_{\mathrm{T}}=0.5 \mathrm{~A}
\end{aligned}
$$

See Table 3:

|  | Resistance | Voltage | Current |
| :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | V 1 | I 1 |
| R 2 | $\mathrm{R} 2=100 \Omega$ | V 2 | I 2 |
| R 3 | $\mathrm{R} 3=50 \Omega$ | V 3 | I 3 |
| Total | $\mathrm{R} \mathrm{T}=100 \Omega$ | $\mathrm{~V}_{\mathrm{T}}=50 \mathrm{~V}$ | $\mathrm{I}_{\mathrm{T}}=\mathbf{0 . 5} \mathbf{~ A}$ |

Table 3
o Expanding the Circuit: Find all remaining values of voltage and current.

- Step 5: Taking total voltage and total current values, go back to last step in the circuit reduction process, (Figure 6), and insert those values where applicable. We can now determine the current through each series resistor in the equivalent circuit in Figure 8.4.


Figure 6

$$
I_{T}=I_{1,2}=I_{3}=0.5 \mathrm{~A}
$$

Enter the value of $I_{3}$ into the solution table (Table 4).

|  | Resistance | Voltage | Current |
| :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | V 1 | I 1 |
| R 2 | $\mathrm{R} 2=100 \Omega$ | V 2 | I 2 |
| R 3 | $\mathrm{R} 3=50 \Omega$ | V 3 | $\mathrm{I} \mathbf{3}=\mathbf{0 . 5} \mathbf{~ A}$ |
| Total | $\mathrm{R}_{\mathrm{T}}=100 \Omega$ | $\mathrm{~V}_{\mathrm{T}}=50 \mathrm{~V}$ | $\mathrm{I}_{\mathrm{T}}=0.5 \mathrm{~A}$ |

## Table 4

- Step 6: From known resistances, total voltage, and total current values from Step 5, use Ohm's Law to calculate unknown values (voltage or current). In our example, calculate the voltage drop $\mathrm{V}_{3}$ across the series resistor $\mathrm{R}_{3}$ and the voltage drop $\mathrm{V}_{1,2}$ across the parallel combination (equivalent resistor, $\mathrm{R}_{1,2}$ ).


Figure 7

$$
\begin{aligned}
& \mathrm{V}_{3}=\mathrm{I}_{3} \times \mathrm{R}_{3} \\
& \mathrm{~V}_{3}=0.5 \mathrm{~A} \times 50 \Omega \\
& \mathrm{~V}_{3}=25 \mathrm{~V} \\
& \mathrm{~V}_{1,2}=\mathrm{I}_{1,2} \times \mathrm{R}_{1,2} \\
& \mathrm{~V}_{1,2}=0.5 \mathrm{~A} \times 50 \Omega \\
& \mathrm{~V}_{1,2}=25 \mathrm{~V}
\end{aligned}
$$

Since $R_{1}$ and $R_{2}$ are parallel resistors, the voltage drops $V_{1}$ and $V_{2}$ are the same value and equal to $V_{1,2}$.

$$
\mathrm{V}_{1}=\mathrm{V}_{2}=\mathrm{V}_{1,2}=25 \mathrm{~V}
$$

We can now enter $\mathrm{V}_{1}, \mathrm{~V}_{2}$, and $\mathrm{V}_{3}$ into the solution table (Table 5).

|  | Resistance | Voltage | Current |
| :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | $\mathbf{V} 1=\mathbf{2 5} \mathrm{V}$ | I 1 |
| R 2 | $\mathrm{R} 2=100 \Omega$ | $\mathbf{V} 2=\mathbf{2 5} \mathrm{V}$ | I 2 |
| R 3 | $\mathrm{R} 3=50 \Omega$ | $\mathrm{~V} 3=\mathbf{2 5} \mathrm{V}$ | $\mathrm{I} 3=0.5 \mathrm{~A}$ |
| Total | $\mathrm{R}_{\mathrm{T}}=100 \Omega$ | $\mathrm{~V}_{\mathrm{T}}=50 \mathrm{~V}$ | $\mathrm{I}_{\mathrm{T}}=0.5 \mathrm{~A}$ |

## Table 5

- Step 7: Repeat steps 5 and 6 until all values for voltage and current are known in the original circuit configuration. Essentially, you will proceed step-by-step from the simplified version of the circuit back into its original, complex form, plugging in values of voltage and current where appropriate until all values of voltage and current are known. Here, calculate the current through each parallel resistor ( $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ ) in the parallel resistor combination. Use Ohm's Law.


Figure 8

$$
\begin{aligned}
& \mathrm{I}_{1}=\mathrm{V}_{1} / \mathrm{R}_{1} \\
& \mathrm{I}_{1}=25 \mathrm{~V} / 100 \Omega \\
& \mathrm{I}_{1}=0.25 \mathrm{~A} \\
& \mathrm{I}_{2}=\mathrm{V}_{2} / \mathrm{R}_{2} \\
& \mathrm{I}_{2}=25 \mathrm{~V} / 100 \Omega \\
& \mathrm{I}_{2}=0.25 \mathrm{~A}
\end{aligned}
$$

Enter $I_{1}$ and $I_{2}$ into the solution table (Table 6).

|  | Resistance | Voltage | Current |
| :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | $\mathrm{~V} 1=25 \mathrm{~V}$ | $\mathbf{I 1}=\mathbf{0 . 2 5 ~ A}$ |
| R 2 | $\mathrm{R} 2=100 \Omega$ | $\mathrm{~V} 2=25 \mathrm{~V}$ | $\mathbf{I 2}=\mathbf{0 . 2 5 ~ A}$ |
| R 3 | $\mathrm{R} 3=50 \Omega$ | $\mathrm{~V} 3=25 \mathrm{~V}$ | $\mathrm{I} 3=0.5 \mathrm{~A}$ |
| Total | $\mathrm{R}_{\mathrm{T}}=100 \Omega$ | $\mathrm{~V}_{\mathrm{T}}=50 \mathrm{~V}$ | $\mathrm{I}_{\mathrm{T}}=0.5 \mathrm{~A}$ |

## Table 6

- Step 8: Calculate power dissipations from known voltage, current, and/or resistance values.

$$
\begin{aligned}
& P_{1}=V_{1} \times \mathrm{I}_{1} \\
& \mathrm{P}_{1}=25 \mathrm{~V} \times 0.25 \mathrm{~A} \\
& \mathrm{P}_{1}=6.25 \mathrm{~W} \\
& \\
& \mathrm{P}_{2}=\mathrm{V}_{2} \times \mathrm{I}_{2} \\
& \mathrm{P}_{2}=25 \mathrm{~V} \times 0.25 \mathrm{~A} \\
& \mathrm{P}_{2}=6.25 \mathrm{~W} \\
& \\
& \mathrm{P}_{3}=\mathrm{V}_{3} \times \mathrm{I}_{3} \\
& \mathrm{P}_{3}=25 \mathrm{~V} \times 0.5 \mathrm{~A} \\
& \mathrm{P}_{3}=12.5 \mathrm{~W} \\
& \\
& \mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{T}} \times \mathrm{I}_{\mathrm{T}} \\
& \mathrm{P}_{\mathrm{T}}=50 \mathrm{~V} \times 0.5 \mathrm{~A} \\
& \mathrm{P}_{\mathrm{T}}=25 \mathrm{~W}
\end{aligned}
$$

The final solution table (Table 7):

|  | Resistance | Voltage | Current | Power |
| :---: | :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | $\mathrm{~V} 1=25 \mathrm{~V}$ | $\mathrm{I} 1=0.25 \mathrm{~A}$ | $\mathrm{P} 1=6.25 \mathrm{~W}$ |
| R 2 | $\mathrm{R} 2=100 \Omega$ | $\mathrm{~V} 2=25 \mathrm{~V}$ | $\mathrm{I} 2=0.25 \mathrm{~A}$ | $\mathrm{P} 2=6.25 \mathrm{~W}$ |
| R 3 | $\mathrm{R} 3=50 \Omega$ | $\mathrm{~V} 3=25 \mathrm{~V}$ | $\mathrm{I} 3=0.5 \mathrm{~A}$ | $\mathrm{P} 3=12.5 \mathrm{~W}$ |
| Total | R | $\mathrm{R}=100 \Omega$ | $\mathrm{~V}_{\mathrm{T}}=50 \mathrm{~V}$ | $\mathrm{I}_{\mathrm{T}}=0.5 \mathrm{~A}$ |

Table 7

## Summary of Values:



Figure 9
o Example 2: This combination circuit is a more complicated set of series and parallel resistors. See Figure 10.


Figure 10
Solve for the all of the unknowns in Table 8.

| Resistor | Resistance | Voltage | Current |
| :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | V 1 | I 1 |
| R 2 | $\mathrm{R} 2=100 \Omega$ | V 2 | I 2 |
| R 3 | $\mathrm{R} 3=50 \Omega$ | V 3 | I 3 |
| R 4 | $\mathrm{R} 4=56 \Omega$ | V 4 | $\mathbf{I 4}$ |
| R 5 | $\mathrm{R} 5=56 \Omega$ | V 5 | I 5 |
| R 6 | $\mathrm{R} 6=150 \Omega$ | V 6 | I 6 |
| Totals | RT | $\mathrm{V}_{\mathrm{T}}=100 \mathrm{~V}$ | $\mathrm{I}_{\mathrm{T}}$ |

## Table 8

o Contracting the Circuit: Finding the Total Resistance, $\mathrm{R}_{\mathrm{T}}$ and Total Current, $\mathrm{I}_{\mathrm{T}}$ :

- Step 1: Assess which resistors in a circuit are connected together in simple series or simple parallel. In this circuit, $\mathrm{R}_{2}$ and $R_{3}$ are in simple series in a parallel branch. Also, $R_{4}$ and $R_{5}$ are in simple parallel configuration.
- Step 2: Re-draw the circuit, (Figure 11), replacing each of those series or parallel resistor combinations identified in Step 1 with a single, equivalent-value resistor.


Figure 11
To determine the equivalent resistance $\mathrm{R}_{2,3}$, add the series resistors $\mathrm{R}_{2}$ and $\mathrm{R}_{3}$ together.

$$
\begin{aligned}
& \mathrm{R}_{2,3}: \\
& \mathrm{R}_{2,3}=\mathrm{R}_{2}+\mathrm{R}_{3} \\
& \mathrm{R}_{2,3}=100 \Omega+50 \Omega \\
& \mathrm{R}_{2,3}=150 \Omega
\end{aligned}
$$

Also determine the equivalent resistor $\mathrm{R}_{4,5}$ for the parallel resistors, $\mathrm{R}_{4}$ and $\mathrm{R}_{5}$, using the formula for parallel resistors of equal value.

$$
\begin{aligned}
\mathrm{R}_{4,5}: & \\
& \mathrm{R}_{4,5}=\mathrm{R} / \mathrm{N} \\
& \mathrm{R}_{4,5}=56 \Omega / 2 \\
& \mathrm{R}_{4,5}=28 \Omega
\end{aligned}
$$

- Step 3: Repeat Steps 1 and 2 until the entire circuit is reduced to one equivalent resistor. (Step 1) - $R_{1}$ and $R_{2,3}$ are in a simple parallel configuration. $\mathrm{R}_{4,5}$ and $\mathrm{R}_{6}$ are in a simple series configuration. (Step 2) - Re-draw the circuit, (Figure 12), with $\mathrm{R}_{1,2,3}$ replacing $\mathrm{R}_{1}$ and $\mathrm{R}_{2,3}$ and $\mathrm{R}_{4,5,6}$ replacing $\mathrm{R}_{4,5}$ and $\mathrm{R}_{6}$.


Figure 12

Determine the equivalent resistor $\mathrm{R}_{1,2,3}$ for the parallel resistors, $R_{1}$ and $R_{2,3}$, using the formula for two parallel resistors.

$$
\begin{aligned}
& \mathrm{R}_{1,2,3}: \\
& \mathrm{R}_{1,2,3}=\mathrm{R}_{1} \times \mathrm{R}_{2,3} / \mathrm{R}_{1}+\mathrm{R}_{2,3} \\
& \mathrm{R}_{1,2,3}=100 \Omega \times 150 \Omega / 100 \Omega+150 \Omega \\
& \mathrm{R}_{1,2,3}=15,000 \Omega^{2} / 250 \Omega \\
& \mathrm{R}_{1,2,3}=60 \Omega
\end{aligned}
$$

Determine the equivalent resistor $\mathrm{R}_{4,5,6}$ for the series resistors, $\mathrm{R}_{4,5}$ and $\mathrm{R}_{6}$, using the formula series resistor formula.

$$
\begin{aligned}
& \mathrm{R}_{4,5,6}: \\
& \mathrm{R}_{4,5,6}=\mathrm{R}_{4,5}+\mathrm{R}_{6} \\
& \mathrm{R}_{4,5,6}=28 \Omega+150 \Omega \\
& \mathrm{R}_{4,5,6}=178 \Omega
\end{aligned}
$$

- Step 3 again: Repeat Steps 1 and 2 until the entire circuit is reduced to one equivalent resistor. (Step 1) - $\mathrm{R}_{1,2,3}$ and $\mathrm{R}_{4,5,6}$ are in a simple series configuration. (Step 2) - Re-draw the circuit, (Figure 13), with $\mathrm{R}_{1,2,3,4,5,6}$ replacing $\mathrm{R}_{1,2,3}$ and $\mathrm{R}_{4.5 .6}$.


Figure 13
Determine the equivalent resistor $R_{1,2,3,4,5,6}$ for the series resistors $R_{1,2,3}$ and $R_{4,5,6}$ using the formula series resistor formula.

$$
\begin{aligned}
& \mathrm{R}_{1,2,3,4,5,6}=\mathrm{R}_{1,2,3}+\mathrm{R}_{4,5,6} \\
& \mathrm{R}_{1,2,3,4,5,6}=60 \Omega+178 \Omega \\
& \mathrm{R}_{1,2,3,4,5,6}=238 \Omega \\
& \mathrm{R}_{1,2,3,4,5,6}=\mathrm{R}_{\mathrm{T}}
\end{aligned}
$$

Enter $\mathrm{R}_{\mathrm{T}}$, $\left(\mathrm{R}_{1,2,3,4,5,6}\right)$, into the solution table (Table 9$)$.

| Resistor | Resistance | Voltage | Current |
| :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | V 1 | I 1 |
| R 2 | $\mathrm{R} 2=100 \Omega$ | V 2 | I 2 |
| R 3 | $\mathrm{R} 3=50 \Omega$ | V 3 | I 3 |
| R 4 | $\mathrm{R} 4=56 \Omega$ | V 4 | I 4 |
| R 5 | $\mathrm{R} 5=56 \Omega$ | V 5 | I 5 |
| R 6 | $\mathrm{R} 6=150 \Omega$ | V 6 | I 6 |
| Totals | $\mathrm{R}_{\mathrm{T}}=\mathbf{2 3 8} \Omega$ | $\mathrm{V}_{\mathrm{T}}=100 \mathrm{~V}$ | I T |

## Table 9

- Step 4: Find the total current for the circuit.

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{T}}=\mathrm{V}_{\mathrm{T}} / \mathrm{R}_{\mathrm{T}} \\
& \mathrm{I}_{\mathrm{T}}=100 \mathrm{~V} / 238 \Omega \\
& \mathrm{I}_{\mathrm{T}}=0.42 \mathrm{~A}
\end{aligned}
$$

See Table 10:

| Resistor | Resistance | Voltage | Current |
| :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | V 1 | I 1 |
| R 2 | $\mathrm{R} 2=100 \Omega$ | V 2 | I 2 |
| R 3 | $\mathrm{R} 3=50 \Omega$ | V 3 | I 3 |
| R 4 | $\mathrm{R} 4=56 \Omega$ | V 4 | I 4 |
| R 5 | $\mathrm{R} 5=56 \Omega$ | V 5 | I 5 |
| R 6 | $\mathrm{R} 6=150 \Omega$ | V 6 | I 6 |
| Totals | $\mathrm{R}_{\mathrm{T}}=238 \Omega$ | $\mathrm{~V}_{\mathrm{T}}=100 \mathrm{~V}$ | $\mathrm{I}_{\mathrm{T}}=\mathbf{0 . 4 2} \mathbf{A}$ |

Table 10
o Expanding the Circuit: Find all remaining values of voltage and current.

- Step 5: Taking total voltage and total current values, go back to last step in the circuit reduction process, (Figure 8.12), and insert those values where applicable.


Figure 14
Determine the current through each series resistor in the equivalent circuit in Figure 14.

$$
I_{T}=I_{1,2,3}=I_{4,5,6}=0.42 \mathrm{~A}
$$

Also, we can calculate the voltage drop $\mathrm{V}_{1,2,3}$ across resistor $\mathrm{R}_{1,2,3}$.

$$
\begin{aligned}
& \mathrm{V}_{1,2,3}=\mathrm{I}_{1,2,3} \times \mathrm{R}_{1,2,3} \\
& \mathrm{~V}_{1,2,3}=0.42 \mathrm{~A} \times 60 \Omega \\
& \mathrm{~V}_{1,2,3}=25.2 \mathrm{~V}
\end{aligned}
$$

- Step 6: From known resistances and total voltage / total current values from Step 5, use Ohm's Law to calculate unknown values (voltage or current). See Figure 15.


Figure 15
$\mathrm{R}_{1}$ and $\mathrm{R}_{2,3}$ :
Since $R_{1}$ and $R_{2,3}$ are parallel resistors, the voltage drops $\mathrm{V}_{1}$ and $V_{2,3}$ are the same value and equal to $\mathrm{V}_{1,2,3}$.

$$
\mathrm{V}_{1}=\mathrm{V}_{2,3}=\mathrm{V}_{1,2,3}=25.2 \mathrm{~V}
$$

Knowing $\mathrm{V}_{1}$ and $\mathrm{R}_{1}$, solve for $\mathrm{I}_{1}$ :

$$
\begin{aligned}
& \mathrm{I}_{1}=\mathrm{V}_{1} / \mathrm{R}_{1} \\
& \mathrm{I}_{1}=25.2 \mathrm{~V} / 100 \Omega \\
& \mathrm{I}_{1}=0.25 \mathrm{~A}
\end{aligned}
$$

The current through $R_{2,3}$ is the total current $I_{T}$ less $I_{1}$.

$$
\begin{aligned}
& \mathrm{I}_{2,3}=\mathrm{I}_{\mathrm{T}}-\mathrm{I}_{1} \\
& \mathrm{I}_{2,3}=0.42 \mathrm{~A}-0.252 \mathrm{~A} \\
& \mathrm{I}_{2,3}=0.168 \mathrm{~A}
\end{aligned}
$$

$I_{2,3}$ could have been solved using Ohm's Law, $I_{2,3}=V_{2,3} / R_{2,3}$
$\mathrm{R}_{4,5}$ and $\mathrm{R}_{6}$ :
$R_{4,5}$ and $R_{6}$ are in series so they have the same current through them as $\mathrm{R}_{4,5,6}$.

$$
I_{4,5,6}=I_{4,5}=I_{6}=0.42 \mathrm{~A}
$$

Knowing $I_{6}$ and $R_{6}$, solve for $V_{6}$ :

$$
\begin{aligned}
& V_{6}=I_{6} \times R_{6} \\
& V_{6}=0.42 \mathrm{~A} \times 150 \Omega \\
& V_{6}=63 \mathrm{~V}
\end{aligned}
$$

Knowing $\mathrm{I}_{4,5}$ and $\mathrm{R}_{4,5}$, solve for $\mathrm{V}_{4,5}$ :

$$
\begin{aligned}
& \mathrm{V}_{4,5}=\mathrm{I}_{4,5} \times \mathrm{R}_{4,5} \\
& \mathrm{~V}_{4,5}=0.42 \mathrm{~A} \times 28 \Omega \\
& \mathrm{~V}_{4,5}=11.8 \mathrm{~V}
\end{aligned}
$$

Update the solutions table with the values of $\mathrm{V}_{1}, \mathrm{I}_{1}, \mathrm{~V}_{6}$ and $\mathrm{I}_{6}$ (Table 11).

| Resistor | Resistance | Voltage | Current |
| :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | $\mathrm{~V} 1=\mathbf{2 5 . 2} \mathrm{V}$ | $\mathbf{I 1}=\mathbf{0 . 2 5 2} \mathbf{A}$ |
| R 2 | $\mathrm{R} 2=100 \Omega$ | V 2 | I |
| R 3 | $\mathrm{R} 3=50 \Omega$ | V 3 | I 3 |
| R 4 | $\mathrm{R} 4=56 \Omega$ | V 4 | I 4 |
| R 5 | $\mathrm{R} 5=56 \Omega$ | V 5 | I |
| R 6 | $\mathrm{R} 6=150 \Omega$ | $\mathrm{~V} 6=\mathbf{6 3} \mathrm{V}$ | $\mathbf{I 6}=\mathbf{0 . 4 2} \mathbf{A}$ |
| Totals | $\mathrm{R} T=238 \Omega$ | $\mathrm{~V}_{T}=100 \mathrm{~V}$ | $\mathrm{I}_{\mathrm{T}}=0.42 \mathrm{~A}$ |

Table 11

- Step 7: Repeat steps 5 and 6 until all values for voltage and current are known in the original circuit configuration.


Figure 16
$\mathrm{R}_{2}$ and $\mathrm{R}_{3}$ :
$R_{2}$ and $R_{3}$ are in series so they have the same current through them as $\mathrm{R}_{2,3}$.

$$
\mathrm{I}_{2,3}=\mathrm{I}_{2}=\mathrm{I}_{3}=0.168 \mathrm{~A}
$$

Knowing $I_{2}$ and $R_{2}$, solve for $V_{2}$ :

$$
\begin{aligned}
& \mathrm{V}_{2}=\mathrm{I}_{2} \times \mathrm{R}_{2} \\
& \mathrm{~V}_{2}=0.168 \mathrm{~A} \times 100 \Omega \\
& \mathrm{~V}_{2}=16.8 \mathrm{~V}
\end{aligned}
$$

Knowing $I_{3}$ and $R_{3}$, solve for $V_{3}$ :

$$
\begin{aligned}
& \mathrm{V}_{3}=\mathrm{I}_{3} \times \mathrm{R}_{3} \\
& \mathrm{~V}_{3}=0.168 \mathrm{~A} \times 50 \Omega \\
& \mathrm{~V}_{3}=8.4 \mathrm{~V}
\end{aligned}
$$

$R_{4}$ and $R_{5}$ :
Since $R_{4}$ and $R_{5}$ are parallel resistors, the voltage drops $\mathrm{V}_{4}$ and $\mathrm{V}_{5}$ are the same value and equal to $\mathrm{V}_{4,5}$.

$$
V_{4}=V_{5}=V_{4,5}=11.8 \mathrm{~V}
$$

Knowing $\mathrm{V}_{4}$ and $\mathrm{R}_{4}$, solve for $\mathrm{I}_{4}$ :

$$
\begin{aligned}
& \mathrm{I}_{4}=\mathrm{V}_{4} / \mathrm{R}_{4} \\
& \mathrm{I}_{4}=11.8 \mathrm{~V} / 56 \Omega \\
& \mathrm{I}_{4}=0.21 \mathrm{~A}
\end{aligned}
$$

Since $R_{4}$ and $R_{5}$ are equal,

$$
I_{4}=I_{5}
$$

Record voltages and currents into the solution table (Table 12).

| Resistor | Resistance | Voltage | Current |
| :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | $\mathrm{~V} 1=25.2 \mathrm{~V}$ | $\mathrm{I} 1=0.252 \mathrm{~A}$ |
| R 2 | $\mathrm{R} 2=100 \Omega$ | $\mathrm{~V} 2=16.8 \mathrm{~V}$ | $\mathrm{I} 2=\mathbf{0 . 1 6 8 ~ A}$ |
| R 3 | $\mathrm{R} 3=50 \Omega$ | $\mathrm{~V} 3=\mathbf{8 . 4 ~ V}$ | $\mathrm{I} 3=\mathbf{0 . 1 6 8 ~ A}$ |
| R 4 | $\mathrm{R} 4=56 \Omega$ | $\mathrm{~V} 4=11.8 \mathrm{~V}$ | $\mathrm{I4}=\mathbf{0 . 2 1} \mathrm{A}$ |
| R 5 | $\mathrm{R} 5=56 \Omega$ | $\mathrm{~V} 5=11.8 \mathrm{~V}$ | $\mathrm{I} 5=\mathbf{0 . 2 1} \mathrm{A}$ |
| R 6 | $\mathrm{R} 6=150 \Omega$ | $\mathrm{~V} 6=63 \mathrm{~V}$ | $\mathrm{I} 6=0.42 \mathrm{~A}$ |
| Totals | $\mathrm{R} T=238 \Omega$ | $\mathrm{~V} T=100 \mathrm{~V}$ | $\mathrm{I}=0.42 \mathrm{~A}$ |

## Table 12

- Step 8: Calculate power dissipations from known voltage, current, and/or resistance values.

$$
\begin{aligned}
& \mathrm{P}_{1}=\mathrm{V}_{1} \times \mathrm{I}_{1} \\
& \mathrm{P}_{1}=25.2 \mathrm{~V} \times 0.252 \mathrm{~A} \\
& \mathrm{P}_{1}=6.3 \mathrm{~W} \\
& \mathrm{P}_{2}=\mathrm{V}_{2} \times \mathrm{I}_{2} \\
& \mathrm{P}_{2}=16.8 \mathrm{~V} \times 0.168 \mathrm{~A} \\
& \mathrm{P}_{2}=2.8 \mathrm{~W} \\
& \mathrm{P}_{3}=\mathrm{V}_{3} \times \mathrm{I}_{3} \\
& \mathrm{P}_{3}=8.4 \mathrm{~V} \times 0.168 \mathrm{~A} \\
& \mathrm{P}_{3}=1.4 \mathrm{~W} \\
& \mathrm{P}_{4}=\mathrm{V}_{4} \times \mathrm{I}_{4} \\
& \mathrm{P}_{4}=11.8 \mathrm{~V} \times 0.21 \mathrm{~A} \\
& \mathrm{P}_{4}=2.5 \mathrm{~W} \\
& \mathrm{P}_{5}=\mathrm{V}_{5} \times \mathrm{I}_{5} \\
& \mathrm{P}_{5}=11.8 \mathrm{~V} \times 0.21 \mathrm{~A} \\
& \mathrm{P}_{5}=2.5 \mathrm{~W} \\
& \mathrm{P}_{6}=\mathrm{V}_{6} \times \mathrm{I}_{6} \\
& \mathrm{P}_{6}=63 \mathrm{~V} \times 0.42 \mathrm{~A} \\
& \mathrm{P}_{6}=26.5 \mathrm{~W} \\
& \mathrm{P}_{\mathrm{T}}=\mathrm{V}_{\mathrm{T}} \times \mathrm{I}_{\mathrm{T}} \\
& \mathrm{P}_{\mathrm{T}}=100 \mathrm{~V} \times 0.42 \mathrm{~A} \\
& \mathrm{P}_{\mathrm{T}}=42 \mathrm{~W}
\end{aligned}
$$

Check power total:

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{T}}=\mathrm{P}_{1}+\mathrm{P}_{2}+\mathrm{P}_{3}+\mathrm{P}_{4}+\mathrm{P}_{5}+\mathrm{P}_{6} \\
& \mathrm{P}_{\mathrm{T}}=6.3 \mathrm{~W}+2.8 \mathrm{~W}+1.4 \mathrm{~W}+2.5 \mathrm{~W}+2.5 \mathrm{~W}+26.5 \mathrm{~W} \\
& \mathrm{P}_{\mathrm{T}}=42.0 \mathrm{~W}
\end{aligned}
$$

The final solution table (Table 13):

| Resistor | Resistance | Voltage | Current | Power |
| :---: | :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | $\mathrm{~V} 1=25.2 \mathrm{~V}$ | $\mathrm{I} 1=0.252 \mathrm{~A}$ | $\mathrm{P} 1=\mathbf{6 . 3} \mathbf{~ W}$ |
| R 2 | $\mathrm{R} 2=100 \Omega$ | $\mathrm{~V} 2=16.8 \mathrm{~V}$ | $\mathrm{I} 2=0.168 \mathrm{~A}$ | $\mathrm{P} 2=\mathbf{2 . 8} \mathbf{~ W}$ |
| R 3 | $\mathrm{R} 3=50 \Omega$ | $\mathrm{~V} 3=8.4 \mathrm{~V}$ | $\mathrm{I} 3=0.168 \mathrm{~A}$ | $\mathrm{P} 3=1.4 \mathbf{~ W}$ |
| R 4 | $\mathrm{R} 4=56 \Omega$ | $\mathrm{~V} 4=11.8 \mathrm{~V}$ | $\mathrm{I} 4=0.21 \mathrm{~A}$ | $\mathrm{P} 4=\mathbf{2 . 5} \mathbf{~ W}$ |
| R 5 | $\mathrm{R} 5=56 \Omega$ | $\mathrm{~V} 5=11.8 \mathrm{~V}$ | $\mathrm{I} 5=0.21 \mathrm{~A}$ | $\mathrm{P} 5=\mathbf{2 . 5} \mathbf{~ W}$ |
| R 6 | $\mathrm{R} 6=150 \Omega$ | $\mathrm{~V} 6=63 \mathrm{~V}$ | $\mathrm{I} 6=0.42 \mathrm{~A}$ | $\mathrm{P} 6=\mathbf{2 6 . 5} \mathbf{~ W}$ |
| Totals | $\mathrm{RT}=238 \Omega$ | $\mathrm{~V}_{\mathrm{T}}=100 \mathrm{~V}$ | $\mathrm{I}_{\mathrm{T}}=0.42 \mathrm{~A}$ | $\mathrm{P}_{\mathrm{T}}=\mathbf{4 2 . 0} \mathbf{~ W}$ |

Table 13

## Summary of Values:



Figure 17
o Example Problem 1:

- Solve for all of the unknowns in the following circuit. Fill in each unknown in the table below the circuit.


|  | Resistance | Voltage | Current | Power |
| :---: | :---: | :---: | :---: | :---: |
| R1 | $\mathrm{R} 1=33 \Omega$ |  |  |  |
| R2 | $\mathrm{R} 2=15 \Omega$ |  |  |  |
| R3 | $\mathrm{R} 3=22 \Omega$ |  |  |  |
| R4 | $\mathrm{R} 4=47 \Omega$ |  |  |  |
| Total |  | $\mathrm{V}_{\mathrm{T}}=24 \mathrm{~V}$ |  |  |

o Remember, to solve series circuit problems, you can use any or all of the following equations:

$$
\begin{aligned}
& R_{T}=R_{1}+R_{2}+R_{3}+\ldots+R_{N} \\
& V_{T}=V_{1}+V_{2}+V_{3}+\ldots+V_{N} \\
& I_{T}=I_{1}=I_{2}=I_{3}=\ldots I_{N} \\
& V=I \times R
\end{aligned}
$$

o Remember, to solve parallel circuit problems, you can use any or all of the following equations:

$$
\begin{aligned}
& 1 / R_{T}=1 / R_{1}+1 / R_{2}+1 / R_{3}+\ldots \ldots \ldots+1 / R_{N} \\
& V_{T}=V_{1}=V_{2}=V_{3}=\ldots . . V_{N} \\
& I_{T}=I_{1}+I_{2}+I_{3}+\ldots+I_{N} \\
& V=I x R
\end{aligned}
$$

o Example Problem 2:

- Fill in the known values then solve for all of the unknowns in the following circuit.


|  | Resistance | Voltage | Current | Power |
| :---: | :---: | :---: | :---: | :---: |
| R1 |  |  |  |  |
| R2 |  |  |  |  |
| R3 |  |  |  |  |
| R4 |  |  |  |  |
| R5 |  |  |  |  |
| R6 |  |  |  |  |
| Total |  |  |  |  |

o Remember, to solve series circuit problems, you can use any or all of the following equations:

$$
\begin{aligned}
& R_{T}=R_{1}+R_{2}+R_{3}+\ldots+R_{N} \\
& V_{T}=V_{1}+V_{2}+V_{3}+\ldots+V_{N} \\
& I_{T}=I_{1}=I_{2}=I_{3}=\ldots I_{N} \\
& V=I \times R
\end{aligned}
$$

o Remember, to solve parallel circuit problems, you can use any or all of the following equations:

$$
\begin{aligned}
& 1 / R_{T}=1 / R_{1}+1 / R_{2}+1 / R_{3}+\ldots \ldots \ldots+1 / R_{N} \\
& V_{T}=V_{1}=V_{2}=V_{3}=\ldots . . V_{N} \\
& I_{T}=I_{1}+I_{2}+I_{3}+\ldots+I_{N} \\
& V=I \times R
\end{aligned}
$$

- You are given a black box with two terminals. The box is known to contain six 1-ohm resistors. Using ohm-meter, you measure the resistance between the terminals to be the following:
- 0.75 Ohms
- 1.5 Ohms
- 2.25 Ohms
- 3 Ohms
- 4.5 Ohms
- 6 Ohms

Determine the configurations of the six resistors inside the box that create each resistance value.

- Example Problem Solutions:
o Example Problem 1:

|  | Resistance | Voltage | Current | Power |
| :---: | :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=33 \Omega$ | $\mathrm{~V} 1=17.48 \mathrm{~V}$ | $\mathrm{I} 1=0.530 \mathrm{~A}$ | $\mathrm{P} 1=9.25 \mathrm{~W}$ |
| R 2 | $\mathrm{R} 2=15 \Omega$ | $\mathrm{~V} 2=6.52 \mathrm{~V}$ | $\mathrm{I} 2=0.435 \mathrm{~A}$ | $\mathrm{P} 2=2.84 \mathrm{~W}$ |
| R 3 | $\mathrm{R} 3=22 \Omega$ | $\mathrm{~V} 3=2.08 \mathrm{~V}$ | $\mathrm{I} 3=0.095 \mathrm{~A}$ | $\mathrm{P} 3=0.20 \mathrm{~W}$ |
| R 4 | $\mathrm{R} 4=47 \Omega$ | $\mathrm{~V} 4=4.44 \mathrm{~V}$ | $\mathrm{I} 4=0.095 \mathrm{~A}$ | $\mathrm{P} 4=0.42 \mathrm{~W}$ |
| Total | $\mathrm{R}=45.3 \Omega$ | $\mathrm{~V}=24 \mathrm{~V}$ | $\mathrm{I}=0.53 \mathrm{~A}$ | $\mathrm{P}_{\mathrm{T}}=12.71 \mathrm{~W}$ |

o Example Problem 2:

|  | Resistance | Voltage | Current | Power |
| :---: | :---: | :---: | :---: | :---: |
| R 1 | $\mathrm{R} 1=100 \Omega$ | $\mathrm{~V} 1=6.66 \mathrm{~V}$ | $\mathrm{I} 1=0.0666 \mathrm{~A}$ | $\mathrm{P} 1=0.443 \mathrm{~W}$ |
| R 2 | $\mathrm{R} 2=200 \Omega$ | $\mathrm{~V} 2=13.31 \mathrm{~V}$ | $\mathrm{I} 2=0.0666 \mathrm{~A}$ | $\mathrm{P} 2=0.886 \mathrm{~W}$ |
| R 3 | $\mathrm{R} 3=100 \Omega$ | $\mathrm{~V} 3=6.66 \mathrm{~V}$ | $\mathrm{I} 3=0.666 \mathrm{~A}$ | $\mathrm{P} 3=0.443 \mathrm{~W}$ |
| R 4 | $\mathrm{R} 4=56 \Omega$ | $\mathrm{~V} 4=4.43 \mathrm{~V}$ | $\mathrm{I} 4=0.079 \mathrm{~A}$ | $\mathrm{P} 4=0.350 \mathrm{~W}$ |
| R 5 | $\mathrm{R} 5=82 \Omega$ | $\mathrm{~V} 5=4.43 \mathrm{~V}$ | $\mathrm{I} 5=0.054 \mathrm{~A}$ | $\mathrm{P} 5=0.239 \mathrm{~W}$ |
| R 6 | $\mathrm{R} 6=47 \Omega$ | $\mathrm{~V} 6=6.26 \mathrm{~V}$ | $\mathrm{I} 6=0.1331 \mathrm{~A}$ | $\mathrm{P} 6=0.833 \mathrm{~W}$ |
| Total | $\mathrm{RT}=180.28 \Omega$ | $\mathrm{~V}=24 \mathrm{~V}$ | $\mathrm{I}_{\mathrm{T}}=0.1331 \mathrm{~A}$ | $\mathrm{P}_{\mathrm{T}}=3.194 \mathrm{~W}$ |

