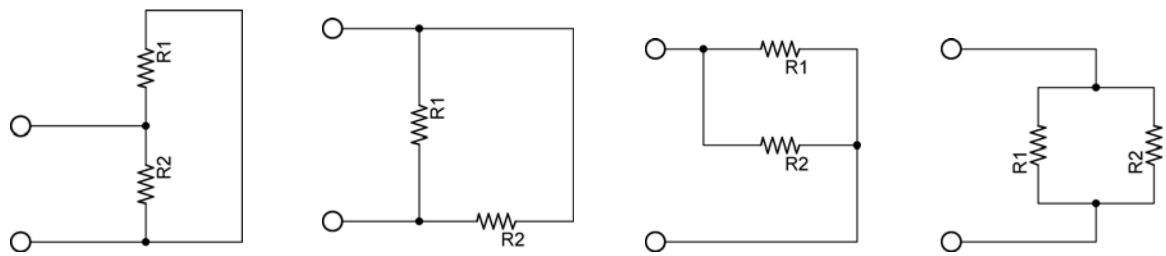
Cornerstone Electronics Technology and Robotics I Week 14
Parallel Circuits

- Administration:
  - Prayer
- Electricity and Electronics, **Section 7.1,** Parallel Circuits:
  - A parallel circuit is one that has more than one pathway for the electrons to flow. Unlike series circuit, when you remove a resistor in a parallel circuit, electrons continue to flow.
  - Identifying parallel circuits: Each example below is a circuit with two parallel paths; all of the circuit configurations are electrically equivalent to each other.



Circuit 1 **Circuit 2** **Circuit 3** **Circuit 4**

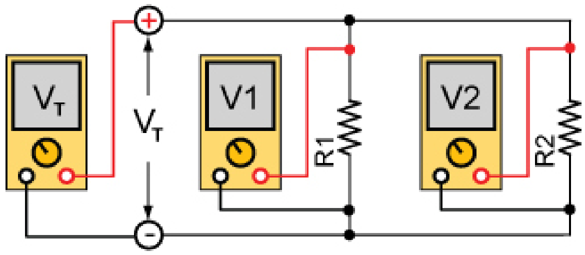
  - Everyday examples of parallel circuits:
    - Electrical outlets in a home See: http://sol.sci.uop.edu/~jfalward/seriesparallelcircuits/seriesparallelcircuits.html (House wiring diagram)
    - Lights in a home
    - Electrical car functions, such as the radio, horn, starter, lights, etc.
  - **Voltage in a Parallel Circuit:**
    - If components are connected in parallel to the source, the voltage drop across each component is the same as the source voltage.
    - Mathematically:

$$V_T = V_1 = V_2 = V_3 = \ldots.. = V_N$$

Where:

V_T = Total voltage applied to the series circuit
V_1 = Voltage drop across R_1
V_2 = Voltage drop across R_2
V_3 = Voltage drop across R_3
V_N = Voltage drop across R_N
N = The number of resistors in the parallel

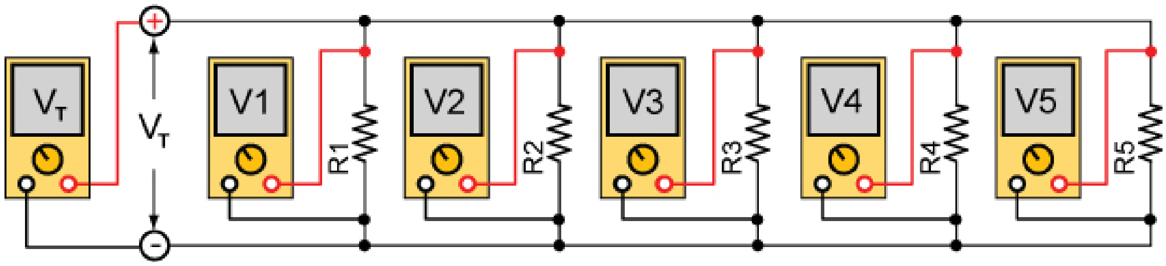
For example, if N = 2, then the 2 resistor parallel circuit and voltmeter setup is as follows:



And the corresponding voltage drop equation is:

$$V_T = V_1 = V_2$$

If N = 5, then the 5 resistor parallel circuit is:



And the corresponding voltage drop equation is:

$$V_T = V_1 = V_2 = V_3 = V_4 = V_5$$

- In the two circuits below, the connections in Figure 1 and Figure 2 are electrically equivalent.

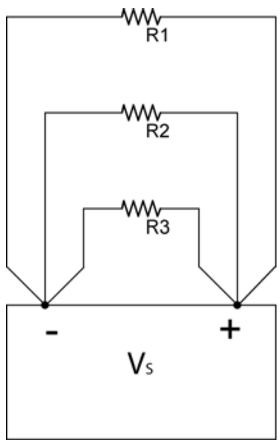


Figure 1

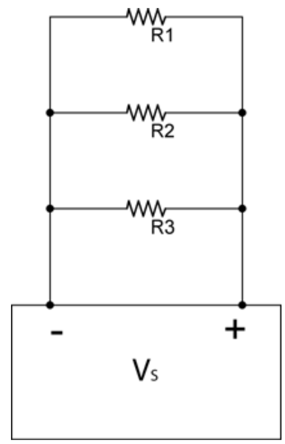


Figure 2

- Perform Parallel Circuits Lab 1 – Voltage Drop in a Parallel Circuit

- **Current in a Parallel Circuit:**
  - Kirchhoff's Current Law: The sum of the currents into a junction is equal to the sum of the currents out of that junction.
  - Mathematically:

    $I_{TOTAL\ in} = I_{TOTAL\ out}$

    Where:

    $I_{TOTAL\ in}$ = the sum of currents into a junction
    $I_{TOTAL\ out}$ = the sum of currents out of a junction

    For Example:

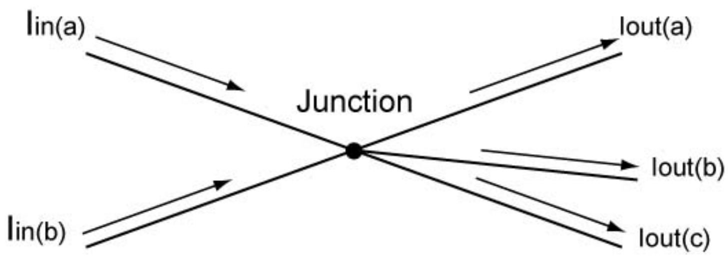


Figure 3

$I_{in(a)} + I_{in(b)} = I_{out(a)} + I_{out(b)} + I_{out(c)}$

Another way of expressing Kirchhoff's Current Law (see Figure 4):

$$I_T = I_1 + I_2 + I_3 + \ldots + I_N$$

Where:

I_T = Total current into a parallel resistor circuit
I_1 = Current through R_1
I_2 = Current through R_2
I_3 = Current through R_3
I_N = Current through R_N

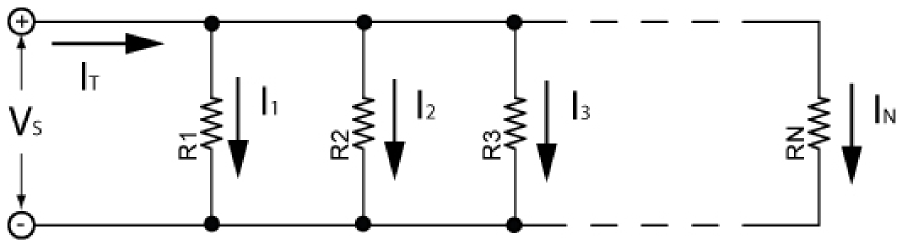
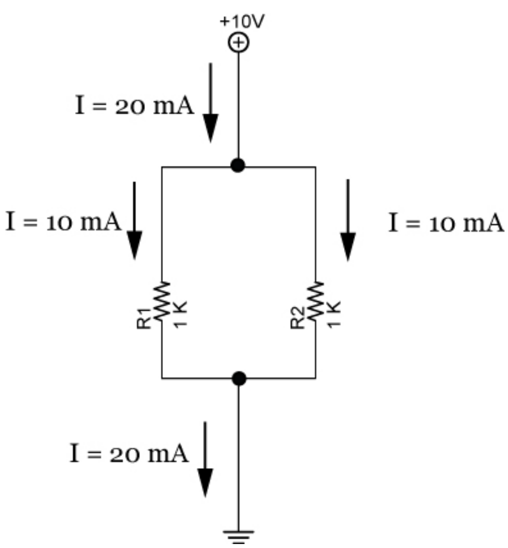


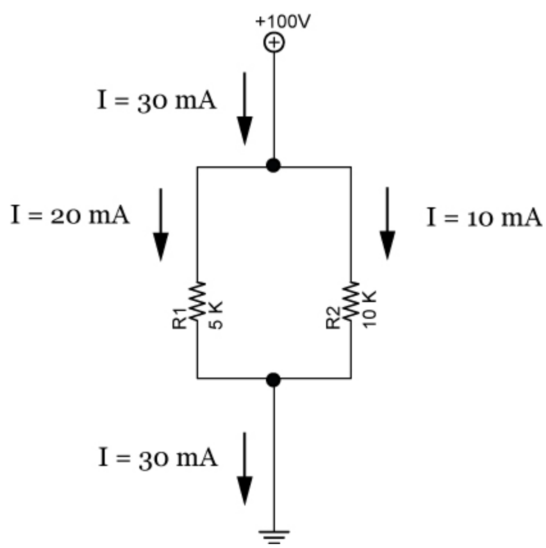
Figure 4

Draw a parallel resistor circuit where N = 4 and show the total current and the currents through each resistor. Assume conventional current flow.

- See applets:
  - http://www.falstad.com/circuit/e-ohms.html
  - http://media.pearsoncmg.com/bc/aw_young_physics_11/pt2a/Media/DCCircuits/1202DCParallel/Main.html (#3, junction law)
- Parallel circuits act as current dividers. See the two examples below.



Example 1



Example 2

- Perform Parallel Circuits Lab 2 – Kirchhoff's Current Law

- **Resistance in Parallel Circuits:**
  - When resistors are connected in parallel circuits, the total resistance is always less than the value of the smallest resistor.
  - Reciprocal Rule:
    - The reciprocal of a number is equal to 1 divided by that number, e.g., the reciprocal of 4 is ¼, and the reciprocal of 87 is 1/87.
    - The total resistance of a parallel circuit is:

    $1/R_T = 1/R_1 + 1/R_2 + 1/R_3 + \ldots\ldots\ldots.+1/R_N$

    Where R_T is the total resistance and
    N is the total number of resistors in parallel.

    - Proof: Since $I_T = I_1 + I_2 + I_3 + \ldots + I_N$:

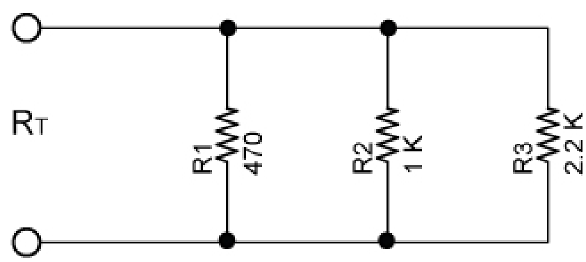
      And $I_T = V_S/R_T$ and $I_1 = V_S/R_1$, $I_2 = V_S/R_2$, etc., then:

      $$V_S/R_T = V_S/R_1 + V_S/R_2 + V_S/R_3 + \ldots.. + V_S/R_N$$

      Now factor out V_S by dividing both sides of the equation by V_S and you arrive at:

      $$1/R_T = 1/R_1 + 1/R_2 + 1/R_3 + \ldots..+1/R_N$$

    - Example: Find the total resistance in Circuit 5:



Circuit 5

$1/R_T = 1/R_1 + 1/R_2 + 1/R_3$
$1/R_T = 1/470 + 1/1000 + 1/2200$
$1/R_T = 0.0021 + 0.001 + 0.0004$
$1/R_T = 0.0035$
$R_T = 1/0.0035$
$R_T = 286\ \Omega$

  - Special Case 1: Two resistor parallel circuit: Since $1/R_T = 1/R_1 + 1/R_2$, then

$$R_T = R_1R_2/R_1+R_2$$

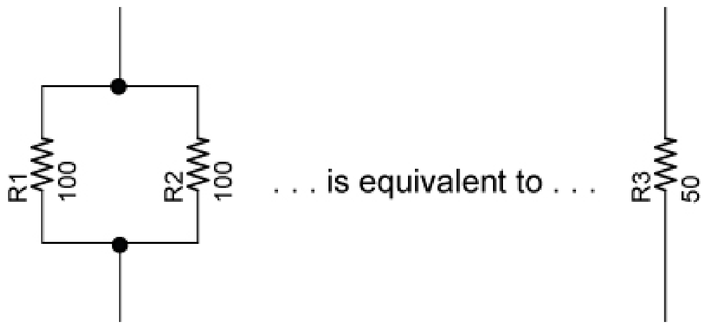
- Special Case 2: Resistors of equal value:

$$R_T = R/N$$

Where:

R = the value of each resistor (all being the same)
N = the number of resistors

For Example:



$$R_T = R/N$$
$$R_T = 100/2$$
$$R_T = 50\ \Omega$$

How many 1 K resistors in parallel would you need to create a total resistance of 1 ohm?

- In a parallel circuit, a resistor that is much smaller than the other resistors dominates.
- Applet: http://www.lon-capa.org/~mmp/kap20/RR506a.htm
- Perform Parallel Circuits Lab 3 – Total Resistance in a Parallel Circuit

o **Power in a Parallel Circuit:**
  - The total power is equal to the sum of all the power of each resistor in the parallel circuit.

$$P_T = P_1 + P_2 + P_3 + \ldots\ldots + P_N$$

Where P_T is the total power consumed in the circuit and
N is the total number of resistors in parallel.

  - Power is also equal to the source voltage times the total current.

$$P_T = V_T \times I_T$$

Where P_T is the total power consumed in the circuit,
V_T is the source voltage, and
I_T is the total current

- Electricity and Electronics, **Section 7.2,** Applications and Troubleshooting Parallel Circuits:
    - Solving for Resistance, Voltage, and Current in a Parallel Resistor Circuits:
        - Four equations are used to solve parallel resistor circuits. They are:

          $$\mathbf{1/R_T = 1/R_1 + 1/R_2 + 1/R_3 + \ldots\ldots\ldots.. + 1/R_N}$$
          $$\mathbf{V_T = V_1 = V_2 = V_3 = \ldots..\ V_N}$$
          $$\mathbf{I_T = I_1 + I_2 + I_3 + \ldots + I_N}$$
          $$\mathbf{V = I \times R}$$

          V = I x R can be applied to the total circuit ($V_T = I_T \times R_T$) and to individual resistors ($V_1 = I_1 \times R_1$).

        - A table will be used to help solve our circuits. To begin, a table as shown in Table 1 corresponds to the circuit in Figure 5:

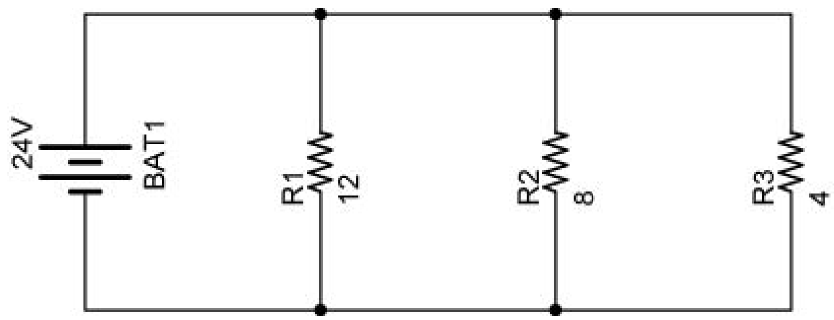


Figure 5

	Resistance	Voltage	Current
R1	R1 = 12 Ω		
R2	R2 = 8 Ω		
R3	R3 = 4 Ω		
Total		V_T = 24 V	

Table 1

- Table 2 lists all of the unknowns that will be solved.

	Resistance	Voltage	Current
R1	R1 = 12 Ω	V1	I1
R2	R2 = 8 Ω	V2	I2
R3	R3 = 4 Ω	V3	I3
Total	R_T	V_T = 24 V	I_T

Table 2

- Step 1: Find V_1, V_2, and V_3.

$$V_T = V_1 = V_2 = V_3 = 24\ V$$

See Table 3:

	Resistance	Voltage	Current
R1	R1 = 12 Ω	V1 = 24 V	I_1
R2	R2 = 8 Ω	V2 = 24 V	I_2
R3	R3 = 4 Ω	V3 = 24 V	I_3
Total	R_T	V_T = 24 V	I_T

Table 3

- Step 2: Find I_1, I_2, and I_3.

$V_1 = I_1 \times R_1$, therefore,
$I_1 = V_1 / R_1$
$I_1 = 24\ V / 12\ \Omega$
$I_1 = 2\ A$

$I_2 = V_2 / R_2$
$I_2 = 24\ V / 8\ \Omega$
$I_2 = 3\ A$

$I_3 = V_3 / R_3$
$I_3 = 24\ V / 4\ \Omega$
$I_3 = 6\ A$

See Table 4:

	Resistance	Voltage	Current
R1	R1 = 12 Ω	V1 = 24 V	I1 = 2 A
R2	R2 = 8 Ω	V2 = 24 V	I2 = 3 A
R3	R3 = 4 Ω	V3 = 24 V	I3 = 6 A
Total	R_T	V_T = 24 V	I_T

Table 4

- Step 3: Find I_T.

$$I_T = I_1 + I_2 + I_3$$
$$I_T = 2\text{ A} + 3\text{ A} + 6\text{ A}$$
$$I_T = 11\text{ A}$$

See Table 5:

	Resistance	Voltage	Current
R1	R1 = 12 Ω	V1 = 24 V	I1 = 2 A
R2	R2 = 8 Ω	V2 = 24 V	I2 = 3 A
R3	R3 = 4 Ω	V3 = 24 V	I3 = 6 A
Total	R_T	V_T = 24 V	I_T = 11 A

Table 5

- Step 5: Find R_T.

$$1/R_T = 1/R_1 + 1/R_2 + 1/R_3$$
$$1/R_T = 1/12 + 1/8 + 1/4$$
$$1/R_T = 0.083 + 0.125 + 0.25$$
$$1/R_T = 0.458$$
$$R_T = 1/0.458$$
$$R_T = 2.18\ \Omega$$

Or an alternate way:

$$V_T = I_T \times R_T\text{, therefore,}$$
$$R_T = V_T / I_T$$
$$R_T = 24\text{ V} / 11\text{ A}$$
$$R_T = 2.18\ \Omega$$

See Table 6:

	Resistance	Voltage	Current
R1	R1 = 12 Ω	V1 = 24 V	I1 = 2 A
R2	R2 = 8 Ω	V2 = 24 V	I2 = 3 A
R3	R3 = 4 Ω	V3 = 24 V	I3 = 6 A
Total	R_T = 2.18 Ω	V_T = 24 V	I_T = 11 A

Table 6

- Since all of the resistances, voltages, and currents are solved in the present problem, the power can now be calculated.

- Solving for Power in a Parallel Resistor Circuits:
  - Two equations are used to solve for power in a parallel resistor circuit. They are:

    $P_T = P_1 + P_2 + P_3 + ... + P_N$
    $P = V \times I$

    $P = V \times I$ can be applied to the total circuit ($P_T = V_T \times I_T$) and to individual resistors ($P_1 = V_1 \times I_1$).
  - A column for power will be added to the table already used to solve our circuit. See Table 7.

	Resistance	Voltage	Current	Power
R1	R1 = 12 Ω	V1 = 24 V	I1 = 2 A	P1
R2	R2 = 8 Ω	V2 = 24 V	I2 = 3 A	P2
R3	R3 = 4 Ω	V3 = 24 V	I3 = 6 A	P3
Total	R_T = 2.18 Ω	V_T = 24 V	I_T = 11 A	P_T

Table 7

  - Step 5: Solve for P_1, P_2, P_3, and P_T.

$P_1 = V_1 \times I_1$
$P_1 = 24\ V \times 2\ A$
$P_1 = 48\ W$

$P_2 = V_2 \times I_2$
$P_2 = 24\ V \times 3\ A$
$P_2 = 72\ W$

$P_3 = V_3 \times I_3$
$P_3 = 24\ V \times 6\ A$
$P_3 = 144\ W$

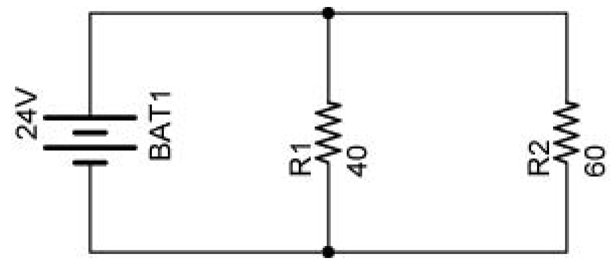
$P_T = P_1 + P_2 + P_3$
$P_T = 48\ W + 72\ W + 144\ W$
$P_T = 264\ W$

See Table 8:

	Resistance	Voltage	Current	Power
R1	R1 = 12 Ω	V1 = 24 V	I1 = 2 A	P1 = 48 W
R2	R2 = 8 Ω	V2 = 24 V	I2 = 3 A	P2 = 72 W
R3	R3 = 4 Ω	V3 = 24 V	I3 = 6 A	P3 = 144 W
Total	R_T = 2.18 Ω	V_T = 24 V	I_T = 11 A	P_T = 264 W

Table 8

- Example Problem 1:
  - In following example problems, insert the known values into the tables and solve for all of the unknowns for the corresponding circuits.

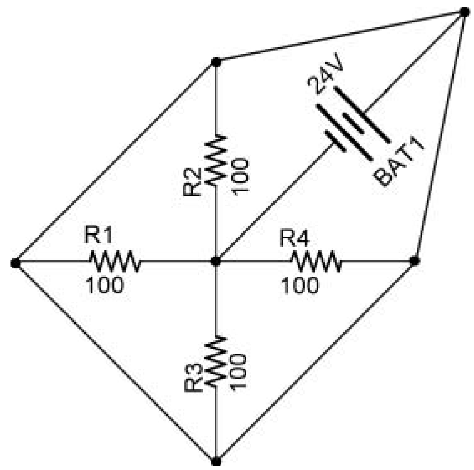


	Resistance	Voltage	Current	Power
R_1				
R_2				
Total				

- Remember:

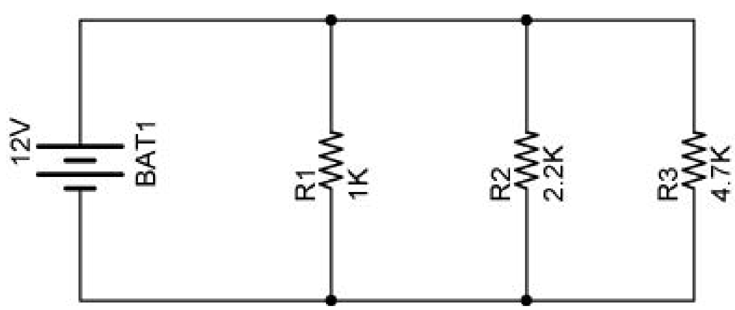
$1/R_T = 1/R_1 + 1/R_2 + 1/R_3 + \ldots\ldots\ldots.. + 1/R_N$
$V_T = V_1 = V_2 = V_3 = \ldots..\ V_N$
$I_T = I_1 + I_2 + I_3 + \ldots + I_N$
$V = I \times R$
$P = V \times I$

- Example Problem 2:



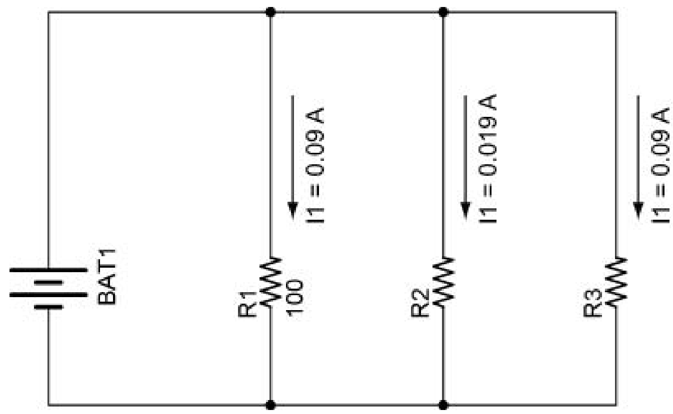
	Resistance	Voltage	Current	Power
R_1				
R_2				
R_3				
R_4				
Total				

- Example Problem 3:



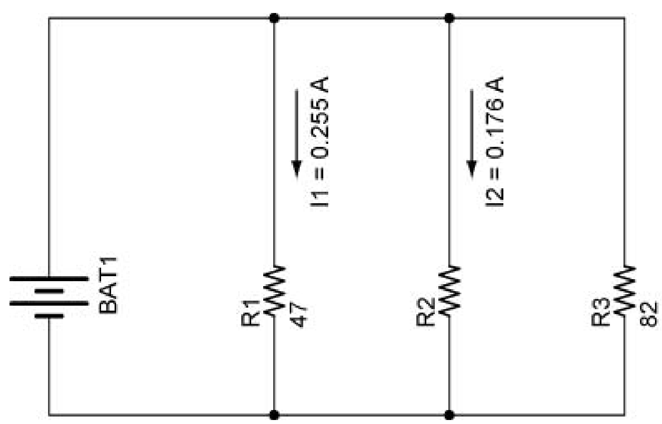
	Resistance	Voltage	Current	Power
R_1				
R_2				
R_3				
Total				

- Example Problem 4:



	Resistance	Voltage	Current	Power
R_1				
R_2				
R_3				
Total				

- Example Problem 5:



	Resistance	Voltage	Current	Power
R_1				
R_2				
R_3				
Total				

- Solve problems 1, 2, 4, and 7 in Student Activity Sheet 7-2.
- Related Web Sites:
  - http://www.physics247.com/solved_problems/basic_circuits.php
  - http://www.acs.ryerson.ca/~kantorek/EES512/tutor2.html
  - http://people.clarkson.edu/~svoboda/eta/designLab/ParallelRDesign.html
  - http://www.glenbrook.k12.il.us/gbssci/phys/Class/circuits/u9l4d.html
  - http://www.allaboutcircuits.com/vol_1/chpt_5/3.html
- Suggested Home-Study Student Activity Sheets 7.1 and 7.2

- Example Problem Solutions:
  - Example Problem 1:

	Resistance	Voltage	Current	Power
R_1	R1 = 40 Ω	V1 = 24 V	I1 = 0.6 A	P1 = 14.4 W
R_2	R2 = 60 Ω	V2 = 24 V	I2 = 0.4 A	P2 = 9.6 W
Total	R_T = 24 Ω	V_T = 24 V	I_T = 1.0 A	P_T = 24.0 W

  - Example Problem 2:

	Resistance	Voltage	Current	Power
R_1	R1 = 100 Ω	V1 = 24 V	I1 = 0.24 A	P1 = 5.76 W
R_2	R2 = 100 Ω	V2 = 24 V	I2 = 0.24 A	P2 = 5.76 W
R_3	R3 = 100 Ω	V3 = 24 V	I3 = 0.24 A	P3 = 5.76 W
R_4	R4 = 100 Ω	V4 = 24 V	I4 = 0.24 A	P4 = 5.76 W
Total	R_T = 25 Ω	V_T = 24 V	I_T = 0.96 A	P_T = 23.04 W

  - Example Problem 3:

	Resistance	Voltage	Current	Power
R_1	R1 = 1K Ω	V1 = 12 V	I1 = 0.012 A	P1 = 0.144 W
R_2	R2 = 2.2K Ω	V2 = 12 V	I2 = 0.0055 A	P2 = 0.066 W
R_3	R3 = 4.7K Ω	V3 = 12 V	I3 = 0.0025 A	P3 = 0.03 W
Total	R_T = 600 Ω	V_T = 12 V	I_T = 0.020 A	P_T = 0.24 W

  - Example Problem 4:

	Resistance	Voltage	Current	Power
R_1	R1 = 100 Ω	V1 = 9V	I1 = 0.09 A	P1 = 0.81 W
R_2	R2 = 484 Ω	V2 = 9 V	I2 = 0.019 A	P2 = 0.171 W
R_3	R3 = 100 Ω	V3 = 9 V	I3 = 0.09 A	P3 = 0.81 W
Total	R_T = 45 Ω	V_T = 9 V	I_T = 0.20 A	P_T = 1.79 W

  - Example Problem 5:

	Resistance	Voltage	Current	Power
R_1	R1 = 47 Ω	V1 = 12 V	I1 = 0.255 A	P1 = 3.06 W
R_2	R2 = 68 Ω	V2 = 12 V	I2 = 0.176 A	P2 = 2.11 W
R_3	R3 = 82 Ω	V3 = 12 V	I3 = 0.146 A	P3 = 1.75 W
Total	R_T = 20.7 Ω	V_T = 12 V	I_T = 0.577 A	P_T = 6.92 W

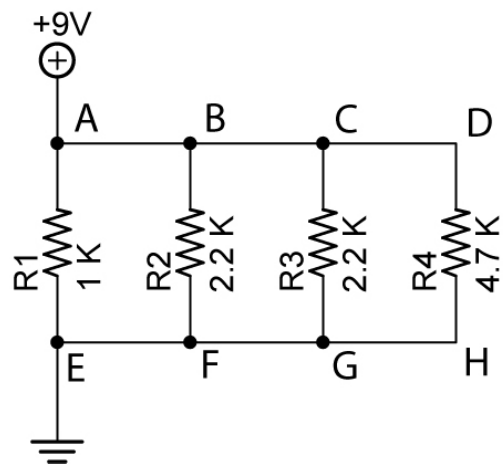
Electronics Technology and Robotics I Week 14
Parallel Circuits Lab 1 – Voltage Drop in a Parallel Circuit

- **Purpose:** The purpose of this lab is to experimentally verify that the voltage drops across parallel resistors are equal.

- **Apparatus and Materials:**

  - 1 – Solderless Breadboard with 9 V Power Supply
  - 1 – Digital Multimeter
  - 1 – 1 K Ohm Resistor
  - 2 – 2.2 K Ohm Resistors
  - 1 – 4.7 K Ohm Resistor

- **Procedure:**
  - Wire the following circuit
  - Measure and record V_{AE}, V_{BF}, V_{CG}, and V_{DH}.



- **Results:**

Points	Voltage Drop
A - E	
B - F	
C - G	
D - H	

- **Conclusions:**
  - How do the voltage drops V_{AE}, V_{BF}, V_{CG}, and V_{DH} relate to each other?

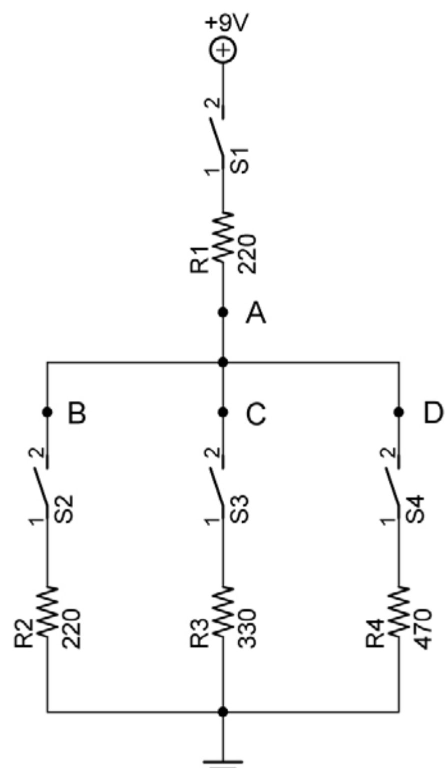
Electronics Technology and Robotics I Week 14
Parallel Circuits Lab 2 – Kirchhoff's Current Law

- **Purpose:** The purpose of this lab is to experimentally verify Kirchhoff's Current Law.

- **Apparatus and Materials:**

  - 1 – Solderless Breadboard with 9 V Power Supply
  - 4 – Digital Multimeters
  - 4 - Switches
  - 2 – 220 Ohm Resistors
  - 1 – 330 Ohm Resistor
  - 1 – 470 Ohm Resistor

- **Procedure:**
  - In the following circuit, simultaneously measure the current at points A, B, C, and D. With all switches closed, see if $I_A = I_B + I_C + I_D$. Record the results. Measure and record the currents of the other combinations in the table using open and closed switches.
  - Verify Kirchhoff's Current Law for each case.



Note how the current through R_1 changes as resistors R_2, R_3, and R_4 are added or removed from the circuit.

- **Results:**

Current In	Current(s) Out			Total of Currents Out
I_A (mA)	I_B (mA)	I_C (mA)	I_D (mA)	Total (mA)
				$I_B + I_C + I_D =$
				$I_B + I_C =$
				$I_B + I_D =$
				$I_C + I_D =$
				$I_B =$
				$I_C =$
				$I_D =$

- **Conclusions:**
  - Does the experiment verify Kirchhoff's Current Law? Explain.

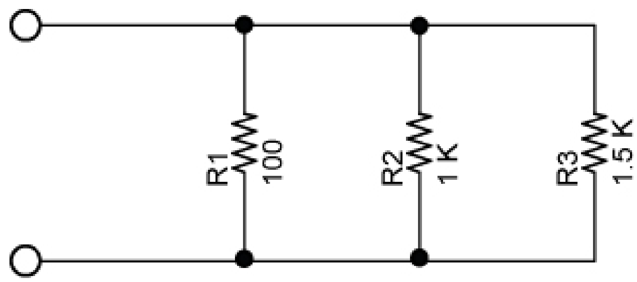
Electronics Technology and Robotics I Week 14
Parallel Circuits Lab 3 – Total Resistance in a Parallel Circuit

- **Purpose:** The purpose of this lab is to experimentally verify the reciprocal rule for total resistance of a parallel circuit.

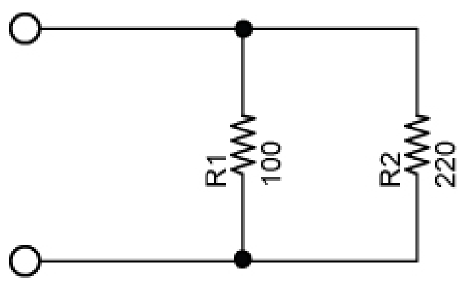
- **Apparatus and Materials:**

  - 1 – Solderless Breadboard
  - 1 – Digital Multimeter
  - 1 – 100 Ohm Resistor
  - 1 – 220 Ohm Resistor
  - 1 – 1K Resistor
  - 3 – 1.5K Resistor

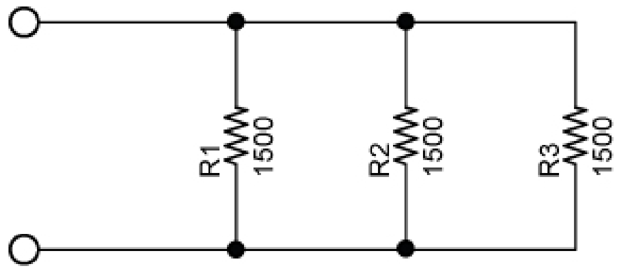
- **Procedure:**
  - Resistors in Parallel:
    - Wire the following circuit below then calculate and measure/record R_T.



  - Two Parallel Resistors:
    - Wire the following circuit below then calculate and measure/record R_T.



- Equal Resistors:
  - Wire the following circuit below then calculate and measure/record R_T.



- **Results:**
  - Resistors in Parallel:

R_T Calculated	R_T Measured

  - Two Parallel Resistors:

R_T Calculated	R_T Measured

  - Equal Resistors:

R_T Calculated	R_T Measured

- **Conclusions:** In each case, evaluate how well the R_T calculated matched the R_T measured. Explain any discrepancies.

  - Resistors in Parallel:

$$1/R_T = 1/R_1 + 1/R_2 + 1/R_3 + \ldots\ldots\ldots\ldots + 1/R_N$$

  - Two Parallel Resistors:

$$R_T = R_1R_2/R_1+R_2$$

- Equal Resistors:

$$R_T = R/N$$